Laborator 1. Calcule numerice şi calcule simbolice în Mathcad şi Mathematica- partea I

Calcul numeric în Mathcad şi Mathematica cu aplicaţii în Algebră

1. $\dfrac{\ln 56 + \sin\cos\sqrt{44.67}}{\sqrt[3]{\ln 678 + \log_{10} 223.98 - tg\sin\dfrac{\pi}{\sqrt{e}}}}$

Mathcad: $\dfrac{\ln(56) + \sin\left(\cos\left(\sqrt{44.67}\right)\right)}{\sqrt[3]{\ln(678) + \log(223.98) - \tan\left(\sin\left(\dfrac{\pi}{\sqrt{e}}\right)\right)}} = 2.4645961$

Mathematica:

```
In[28]:= a = (Log[56] + Sin[Cos[√44.67]]) / ∛(Log[678] + Log10[223.98] - Tan[Sin[π / √e]]) ;

In[29]:= SetPrecision[a, 8]
Out[29]= 2.4645961
```

2. $\sqrt{2}^{\sqrt[3]{3}^{\sqrt[4]{4}^{\sqrt[5]{5}}}}$

Mathcad: $\left[\left[\left(\sqrt{2}\right)^{\sqrt[3]{3}}\right]^{\sqrt[4]{4}}\right]^{\sqrt[5]{5}} = 2.6520023336841$

Mathematica:

```
In[36]:= b = √2^(3^(1/3))^(4^(1/4))^(5^(1/5)) ;

In[40]:= SetPrecision[b, 14]
Out[40]= 2.6520023336841
```

3. $\left(\dfrac{1+i}{1-i}\right)^3 + \left(\dfrac{1-i}{1+i}\right)^3$

Mathcad: $\left(\dfrac{1+i}{1-i}\right)^3 - \left(\dfrac{1-i}{1+i}\right)^3 = -2i$

Mathematica:

```
In[42]:= ((1 + i) / (1 - i))^3 - ((1 - i) / (1 + i))^3
Out[42]= -2 i
```

4. Să se calculeze expresiile următoare:

a) $C = \cos^4 x + \cos^4 3x + \cos^4 5x + \cos^4 7x$, pentru $x = \dfrac{\pi}{8}$;

$$x := \frac{\pi}{8} \qquad i := 1,3..7$$

Mathcad: $C := \sum_i \cos(i \cdot x)^4 \qquad C = 1.5$

sau

$$C := \cos(x)^4 + \cos(3 \cdot x)^4 + \cos(5 \cdot x)^4 + \cos(7 \cdot x)^4$$

Mathematica:

```
In[43]:= x = π / 8;

In[47]:= N[Sum[Cos[k * x]^4, {k, 1, 7, 2}]]

Out[47]= 1.5
```

b) $A = \log_6 7 - \log_6 \frac{7}{36} + \log_2 5 + 3^7 + e^3$

Mathcad: $A := \log(7,6) - \log\left(\frac{7}{36},6\right) + \log(5,2) + 3^7 + e^3$

$A = 2211.407$

Mathematica:

```
In[52]:= N[Log[6, 7] - Log[6, 7 / 36] + Log[2, 5] + 3^7 + e^3, 7]

Out[52]= 2211.407
```

5. Verificaţi dacă numerele 87 şi 41 sunt prime între ele.

Mathcad: $\gcd(87,41) = 1$

Mathematica:

```
In[53]:= GCD[87, 41]

Out[53]= 1
```

Deoarece $c.m.m.d.c(87,41) = 1$ rezultă că numerele 87 şi 41 sunt prime între ele.

6. Aflaţi cel mai mic multiplu comun al numerelor : 40, 36, 126.

Mathcad: $\mathrm{lcm}(40,36,126) = 2520$

Mathematica:

```
In[54]:= LCM[40, 36, 126]

Out[54]= 2520
```

7. Să se calculeze produsul scalar şi cosinusul unghiului dintre vectorii

$$\vec{a} = 2\vec{i} + \vec{j} - \vec{k} \text{ şi } \vec{x} = \frac{1}{2}\vec{a}.$$

$$a := (2 \quad 1 \quad -1)^T \qquad x := \frac{a}{2} \qquad s := a^T \cdot x$$

Mathcad:

$$u := \frac{s}{\left[\sum_{i=0}^{2} (a_i)^2\right]^{\frac{1}{2}} \cdot \left[\sum_{i=0}^{2} (x_i)^2\right]^{\frac{1}{2}}} \qquad u = (1)$$

Mathematica:

```
In[56]:= {2, 1, -1}.{1, 1/2, -1/2}
Out[56]= 3

In[57]:= Dot[{2, 1, -1}, {1, 1/2, -1/2}]
Out[57]= 3

In[58]:= VectorAngle[{2, 1, -1}, {1, 1/2, -1/2}]
Out[58]= 0
```

8. Calculaţi produsul vectorial al vectorilor $\vec{u} = 3\vec{i} - 2\vec{j} + \vec{k}$, $\vec{v} = -\vec{j} + 4\vec{k}$.

$$u := (3 \quad -2 \quad 1)^T \qquad v := (0 \quad -1 \quad 4)^T$$

Mathcad:

$$u \times v = \begin{pmatrix} -7 \\ -12 \\ -3 \end{pmatrix}$$

Mathematica:

```
In[3]:= {3, -2, 1} × {0, -1, 4}
Out[3]= {-7, -12, -3}
```

9. Calculaţi produsul mixt al vectorilor: $\vec{u} = \vec{i} + 2\vec{j} - \vec{k}$, $\vec{v} = 2\vec{i} + 3\vec{j} + 4\vec{k}$, $\vec{w} = -4\vec{i} + 3\vec{j} - 2\vec{k}$.

Mathcad:

$$A := \begin{pmatrix} 1 & 2 & -1 \\ 2 & 3 & 4 \\ -4 & 3 & -2 \end{pmatrix} \qquad |A| = -60$$

Mathematica:

```
In[59]:= A = {{1, 2, -1}, {2, 3, 4}, {-4, 3, -2}};

In[60]:= Det[A]
Out[60]= -60
```

10. Determinati vectorii si valorile proprii ai matricei $A = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 2 & -3 \\ 1 & -1 & 0 \end{pmatrix}$.

Mathcad:

$$A := \begin{pmatrix} 1 & 0 & 0 \\ 1 & 2 & -3 \\ 1 & -1 & 0 \end{pmatrix}$$

$$\text{eigenvals}(A) = \begin{pmatrix} -1 \\ 3 \\ 1 \end{pmatrix} \qquad \text{eigenvecs}(A) = \begin{pmatrix} 0 & 0 & 0.816 \\ 0.707 & 0.949 & 0.408 \\ 0.707 & -0.316 & 0.408 \end{pmatrix}$$

Mathematica:

```
In[63]:= A = {{1, 0, 0}, {1, 2, -3}, {1, -1, 0}}
Out[63]= {{1, 0, 0}, {1, 2, -3}, {1, -1, 0}}

In[64]:= Eigenvalues[A]
Out[64]= {3, -1, 1}

In[65]:= Eigenvectors[A]
Out[65]= {{0, -3, 1}, {0, 1, 1}, {2, 1, 1}}
```

11. Determinaţi vectorul propriu coresponzător celei mai mari valori proprii (in valoare absoluta) a matricei

$$A=\begin{pmatrix} 3 & 4 & -1.6 & 0 & 1 \\ 0 & -1 & 0.5 & 6 & 0 \\ 0 & 0 & -7.6 & 5 & 8 \\ 6 & 4 & 3 & 20 & -1 \\ 8 & 0 & -0.6 & 3 & 0 \end{pmatrix}.$$

Se foloseşte funcţia *eigenvec*(A, v_i), unde v este vectorul ce conţine valorile proprii ale lui A iar i este indicele celei mai mari valori proprii.

Calcul numeric în Mathcad şi Mathematica cu aplicaţii în Analiză matematică

1) Calculaţi derivatele de mai jos, în punctele indicate:

a) $f(x)=\arcsin\dfrac{2\sqrt{x}}{1+x}$, $f''(5.7)=?$; b) $f(x)=2^{x^2-2x}$, $f^{(3)}(-0.2)=?$

Mathcad:

$f(x) := 2^{x^2-2\cdot x}$

$x := -0.2$ $\quad \dfrac{d^3}{dx^3}f(x) = -15.631$

Mathematica:

```
In[82]:= f[u_] := 2^(u^2-2*u)

In[84]:= D[f[u], {u, 3}] /. u → -0.2
Out[84]= -15.6311
```

2) Calculaţi derivatele parţiale de ordinul al doilea ale funcţiilor următoare în punctele indicate:

a. $f(x,y)=2x^3y-e^{x^2}$, $(-1,1)$

Mathcad:

$f(x,y) := 2\cdot x^3\cdot y - e^{x^2}$ $\quad x := -1 \quad y := 1$

$\dfrac{\partial^2}{\partial x^2}f(x,y) = -28.31$ $\quad \dfrac{\partial^2}{\partial y^2}f(x,y) = 0$ $\quad \dfrac{\partial}{\partial x}\left(\dfrac{\partial}{\partial y}f(x,y)\right) = 6$

Mathematica:

```
In[4]:= g[u_, v_] := 2*u^3*v - e^u^2

In[8]:= N[∂u,u g[u, v] /. {u → -1, v → 1}, 4]
Out[8]= -28.31

In[9]:= N[∂v,v g[u, v] /. {u → -1, v → 1}, 4]
Out[9]= 0

In[10]:= N[∂u,v g[u, v] /. {u → -1, v → 1}, 4]
Out[10]= 6.000
```

b. $f(x,y)=\sqrt[3]{x^2y}$, $(-2,2)$; c. $f(x,y,z)=xe^{yz}$, $(1,1,1)$.

3) Calculaţi următoarele integrale simple

a) $\int_0^1 x^2\sqrt{x^2+1}\,dx$;

b) $\int_0^{\frac{\pi}{2}} \frac{\sin 2x}{1+\sin^2 x}\,dx$

Mathcad: $\int_0^{\frac{\pi}{2}} \frac{\sin(2\cdot x)}{1+\sin(x)^2}\,dx = 0.693$

Mathematica:

```
In[12]:= N[∫_0^(π/2) Sin[2 * x] / (1 + Sin[x]^2) dx, 4]

Out[12]= 0.6931
```

4) Calculaţi valoarea următoarelor integrale improprii

a) $\int_{-1}^{1} \frac{dx}{(x+1)^{2/3}}$;

b) $\int_0^1 \frac{dx}{\sqrt{x(1-x)}}$

c) $\int_{-\infty}^{\infty} \frac{dx}{1+x^2}$

Mathcad: $\int_{-\infty}^{\infty} \frac{1}{1+x^2}\,dx = 3.142$

Mathematica:

```
In[14]:= N[∫_-∞^∞ 1 / (1 + x^2) dx, 4]

Out[14]= 3.142
```

5) Calculaţi următoarele integrale duble:

a) $\int_0^{\frac{\pi}{2}}\int_0^{\frac{\pi}{2}} \frac{\cos y}{1+\sin x\sin y}\,dx\,dy$;

b) $\int_1^2\int_1^2 \sqrt{\frac{x}{y^3}}\,dx\,dy$.

Calcul simbolic în Mathcad cu aplicaţii în Algebră

1. Simplificaţi expresia $E = \frac{3^{n+2}\cdot 5^n + 3^n\cdot 5^{n+1}}{3^{n+1}\cdot 5^n + 2\cdot 3^{n+1}\cdot 5^n}$.

Mathcad: $\frac{3^{n+2}\cdot 5^n + 3^n\cdot 5^{n+1}}{3^{n+1}\cdot 5^n + 2\cdot 3^{n+1}\cdot 5^n}$ simplify $\rightarrow \frac{14}{9}$

Mathematica:

```
In[15]:= Simplify[(3^(n+2) * 5^n + 3^n * 5^(n+1)) / (3^(n+1) * 5^n + 2 * 3^(n+1) * 5^n)]

Out[15]= 14/9
```

2. Calculaţi suma $S_n = 2 + \frac{3}{2} + \frac{5}{4} + \ldots + \left(1 + \frac{1}{2^{n-1}}\right), n \in \mathrm{N}$.

Mathcad: $\sum_{k=1}^{n} \left(1 + \frac{1}{2^{k-1}}\right)$ simplify $\rightarrow n + 2 - 2^{(1-n)}$

Mathematica:

```
In[17]:= Simplify[Sum[(1 + 1/2^(k-1)), {k, 1, n}]]
Out[17]= 2 - 2^(1-n) + n
```

3. Calculaţi determinantul, transpusa şi inversa simbolică pentru următoarea matrice:

$$A = \begin{pmatrix} \cos x & -\sin x \\ \sin x & \cos x \end{pmatrix}.$$

Mathcad:

$$A(x) := \begin{pmatrix} \cos(x) & -\sin(x) \\ \sin(x) & \cos(x) \end{pmatrix}$$

$$|A(x)| \rightarrow \cos(x)^2 + \sin(x)^2 \text{ simplify } \rightarrow 1 \qquad A(x)^T \rightarrow \begin{pmatrix} \cos(x) & \sin(x) \\ -\sin(x) & \cos(x) \end{pmatrix}$$

$$A(x)^{-1} \rightarrow \begin{bmatrix} \frac{\cos(x)}{\left(\cos(x)^2 + \sin(x)^2\right)} & \frac{\sin(x)}{\left(\cos(x)^2 + \sin(x)^2\right)} \\ \frac{-\sin(x)}{\left(\cos(x)^2 + \sin(x)^2\right)} & \frac{\cos(x)}{\left(\cos(x)^2 + \sin(x)^2\right)} \end{bmatrix}$$

Mathematica:

```
In[18]:= A = {{Cos[x], -Sin[x]}, {Sin[x], Cos[x]}}
Out[18]= {{Cos[x], -Sin[x]}, {Sin[x], Cos[x]}}

In[20]:= Simplify[Det[A]]

In[21]:= Transpose[A]
Out[21]= {{Cos[x], Sin[x]}, {-Sin[x], Cos[x]}}

In[23]:= Simplify[Inverse[A]]
Out[23]= {{Cos[x], Sin[x]}, {-Sin[x], Cos[x]}}
```

4. Să se factorizeze expresia $E = x^3 + yx^2 - 2\sqrt{3}x^2y - 2\sqrt{3}xy^2 + 3y^2x + 3y^3$.

Mathcad: $x^3 + y \cdot x^2 - 2 \cdot \sqrt{3} \cdot x^2 \cdot y - 2 \cdot \sqrt{3} \cdot x \cdot y^2 + 3 \cdot y^2 \cdot x + 3 \cdot y^3$ factor $\rightarrow (x + y) \cdot \left(x - \sqrt{3} \cdot y\right)^2$

Mathematica:

```
In[36]:= Factor[x^3 + y*x^2 - 2*Sqrt[3]*x^2*y - 2*Sqrt[3]*y^2*x + 3*y^2*x + 3*y^3, Extension -> Automatic]
Out[36]= (x + y) (-x + Sqrt[3] y)^2
```

5. Să se dezvolte determinantul $\begin{vmatrix} a+b & -a+b-c & b+c \\ a-b-c & a+b & a+c \\ a+c & b+c & -a-b+c \end{vmatrix}$ iar rezultatul să fie pus sub formă de produs.

6. Colectaţi coficienţii expresiei $xy + x^2y^3 + x^3y + y - y^2$ în raport cu variabila y.

Mathcad: $x \cdot y + x^2 \cdot y^3 + x^3 \cdot y + y - y^3$ collect, $y \rightarrow \left(x^2 - 1\right) \cdot y^3 + \left(x + x^3 + 1\right) \cdot y$

Mathematica:

```
In[38]:= Collect[x * y + x^2 * y^3 + x^3 * y + y - y^3, y]
Out[38]= (1 + x + x^3) y + (-1 + x^2) y^3
```

7. Expandaţi expresia: $E=\left(x^2\sqrt{2}-x\sqrt{3}+\sqrt{5}\right)\left(x^2\sqrt{2}+x\sqrt{3}+\sqrt{5}\right)-\left(x^2\sqrt{2}+\sqrt{5}\right)^2$.

Mathcad: $\left(x^2\cdot\sqrt{2}-x\cdot\sqrt{3}+\sqrt{5}\right)\cdot\left(x^2\cdot\sqrt{2}+x\cdot\sqrt{3}+\sqrt{5}\right)-\left(x^2\cdot\sqrt{2}+\sqrt{5}\right)^2 \text{ expand } \rightarrow -3\cdot x^2$

Mathematica:

```
In[40]:= Expand[(x^2 * Sqrt[2] - x * Sqrt[3] + Sqrt[5]) * (x^2 * Sqrt[2] + x * Sqrt[3] + Sqrt[5]) - (x^2 * Sqrt[2] + Sqrt[5])^2]
Out[40]= -3 x^2
```

8. Se dau polinoamele $P(x)=x^6-x^5-3x^4+x^3-4x-3$, $Q(x)=2x^4-6x^2-8x-6$, $x\in\Re$.

Să se transforme fracţia $F(x)=\dfrac{P(x)}{Q(x)}$ într-o fracţie ireductibilă.

Mathcad: $\dfrac{x^6-x^5-3\cdot x^4+x^3-4\cdot x-3}{2\cdot x^4-6\cdot x^2-8\cdot x-6} \text{ convert , parfrac , x } \rightarrow \dfrac{1}{2}\cdot x^2-\dfrac{1}{2}\cdot x+\dfrac{(1+2\cdot x)}{\left(2\cdot x^2+2\cdot x+2\right)}$

Mathematica:

```
In[47]:= Apart[(x^6 - x^5 - 3 * x^4 + x^3 - 4 * x - 3) / (2 * x^4 - 6 * x^2 - 8 * x - 6), x]
Out[47]= -x/2 + x^2/2 + (1 + 2 x)/(2 (1 + x + x^2))
```

9. Să se determine coeficienţii polinomului $P(x)=\left(x\sqrt{3}-2\right)\left(x\sqrt{3}+2\right)$.

Mathcad: $\left(x\cdot\sqrt{3}-2\right)\cdot\left(x\cdot\sqrt{3}+2\right) \text{ coeffs , x } \rightarrow \begin{pmatrix}-4\\0\\3\end{pmatrix}$

Mathematica:

```
In[48]:= CoefficientList[(x * Sqrt[3] - 2) * (x * Sqrt[3] + 2), x]
Out[48]= {-4, 0, 3}
```

10. Să se afişeze sub formă de număr complex: $(z+2i)^2(z+3i)^3$.

Mathcad: $(z+2i)^2\cdot(z+3i)^3 \text{ complex } \rightarrow z^5-67\cdot z^3+216\cdot z+1i\cdot\left(13\cdot z^4-171\cdot z^2+108\right)$

Mathematica:

```
In[54]:= ComplexExpand[(z + 2 * I)^2 * (z + 3 * I)^3]
Out[54]= 216 z - 67 z^3 + z^5 + I (108 - 171 z^2 + 13 z^4)
```

11. Să se evalueze expresia următoare $\dfrac{|x-1|+|x-3|}{|x-2|}$ pentru $x\le 1$.

Mathcad: $\dfrac{|x-1|+|x-3|}{|x-2|} \text{ assume, x} \le 1 \rightarrow \dfrac{(4-2\cdot x)}{(2-x)} \text{ simplify} \rightarrow 2$

Mathematica:

```
In[56]:= Assuming[x <= 1, Simplify[(Abs[x - 1] + Abs[x - 3]) / Abs[x - 2]]]
Out[56]= 2
```

Laborator 2. Calcule numerice şi calcule simbolice în Mathcad și Mathematica- partea a II-a

Calcul simbolic în Mathcad și Mathematica cu aplicaţii în Analiză matematică

1. Calculaţi următoarele limite:

a) $\lim_{n\to\infty} \sqrt[n]{\frac{(2n)!}{(n!)^2}}$

Mathcad $\lim_{n\to\infty} \sqrt[n]{\frac{(2\cdot n)!}{(n!)^2}} \to 4$

Mathematica

```
In[1]:= Limit[Sqrt[n, (2 * n) ! / (n ! )^2], n → ∞]
Out[13]= 4
```

b) $\lim_{n\to\infty} \frac{1}{n+n^2}(\sin 1+\ldots+\sin n)=\lim_{n\to\infty} \frac{1}{n+n^2}\cdot\sum_{k=1}^{n}\sin k$

Mathcad $\lim_{n\to\infty} \frac{1}{n+n^2}\cdot\sum_{k=1}^{n}\sin(k) \to 0$

Mathematica

```
In[13]:= Limit[1 / (n + n^2) * Sum[Sin[k], {k, 1, n}], n → ∞]
Out[13]= 0
```

c) $\lim_{x\to 0,\, x<0} e^{\frac{1}{x}}$

Mathcad $\lim_{x\to 0^-} e^{\frac{1}{x}} \to 0$

Mathematica

```
In[14]:= Limit[e^(1/x), x -> 0, Direction → 1]
Out[14]= 0
```

d) $\lim_{x\to 0,\, x>0} e^{\frac{1}{x}}$

Mathcad $\lim_{x\to 0^+} e^{\frac{1}{x}} \to \infty$

Mathematica

```
In[15]:= Limit[e^(1/x), x -> 0, Direction → -1]
Out[15]= ∞
```

2. Să se determine raza de convergenţă pentru următoarea serie de puteri:

a) $\sum_{n\geq 1} n^{\ln^2 n}\cdot x^n$

b) $\sum_{n\geq 1}(-1)^n \frac{n+1}{n^2+n+1}x^n$

Mathcad $a(n) := (-1)^n \cdot \frac{n+1}{n^2+n+1}$ $\frac{1}{\lim_{n\to\infty}\left|\frac{a(n+1)}{a(n)}\right|} \to 1$

Mathematica

```
In[4]:= a[n_] := (-1)^n * (n + 1)/(n^2 + n + 1);

In[5]:= 1/Limit[Abs[a[n+1]/a[n]], n → ∞]

Out[5]= 1
```

Pentru a determina raza de convergenţă a unei serii de puteri $\sum_{n\geq 0} a_n x^n$ putem vom folosi una din formulele: $R = \frac{1}{\lim_{n\to\infty}\sqrt[n]{|a_n|}}$ sau $R = \frac{1}{\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right|}$.

3. Calculaţi suma seriei:

a) $\sum_{n\geq 1}\ln\left(1+\frac{1}{n}\right)$;

b) $\sum_{n\geq 1}\frac{2n+1}{n^2(n+1)^2}$;

c) $\sum_{n\geq 1}\frac{n^2+n-1}{(n+1)!}$

Mathcad $\sum_{n=1}^{\infty} \frac{n^2+n-1}{(n+1)!} \to 2$

Mathematica

```
In[6]:= Sum[(n^2 + n - 1)/(n + 1)!, {n, 1, ∞}]

Out[6]= 2
```

4. Calculaţi produsul $\frac{1}{2}\cdot\frac{7}{8}\cdot\frac{17}{18}\cdot\frac{31}{32}\cdot\ldots = \prod_{k\geq 1}\left(1-\frac{1}{2k^2}\right)$

Mathcad $\prod_{k=1}^{\infty}\left(1-\frac{1}{2\cdot k^2}\right) \to \frac{1}{\pi}\cdot\sin\left(\frac{1}{2}\cdot\pi\cdot\sqrt{2}\right)\cdot\sqrt{2}$

Mathematica

```
In[10]:= N[Product[1 - 1/(2 * k^2), {k, 1, ∞}], 4]

Out[10]= 0.3582
```

5. Să se calculeze următoarele derivate:

a) $f(x) = \cos^2\sqrt{x}$; $f'(x) = ?$; b) $f(x) = x\cdot\operatorname{arctg} x - \ln(1+x^2)$, $f''(x) = ?$

c) $f(x)=\dfrac{1}{x^3+6x^2+11x+6}$, $f^{(11)}(x)=?$

6. Să se calculeze derivatele parţiale de ordinul întâi şi al doilea pentru funcţia

a) $f(x,y)=\operatorname{arctg}\dfrac{x+y}{1-xy}$; b) $f(x,y,z)=y^{x^z}$, $x,y>0$

7. Scrieţi primii şapte termeni din dezvoltarea în serie de puteri a funcţiei $f(x)=\dfrac{1}{2}\ln\dfrac{1+x}{1-x}$, $x\in(-1,1)$.

Mathcad $f(x):=\frac{1}{2}\cdot\ln\left(\frac{1+x}{1-x}\right)$ $f(x)\ \text{series},x=0,7\ \rightarrow x+\frac{1}{3}\cdot x^3+\frac{1}{5}\cdot x^5$

Mathematica

```
In[12]:= Series[1/2 * Log[(1 + x)/(1 - x)], {x, 0, 6}]
Out[12]= x + x^3/3 + x^5/5 + O[x]^7
```

8. Calculaţi

a) $\int x^{-4}\left(1-x^2\right)^{\frac{3}{2}}\,\mathrm{d}\,x$;

b) $\int\dfrac{x^2}{(x\cos x-\sin x)^2}\,\mathrm{d}\,x$;

c) $\int\dfrac{\sqrt[3]{1+\sqrt[4]{x}}}{\sqrt{x}}\,\mathrm{d}\,x$;

d) $\int\sqrt{\dfrac{x^2-x+1}{x^2+x+1}}\left(1-\dfrac{1}{x^2}\right)\mathrm{d}\,x$

e) $\int\dfrac{1}{x^2}\cdot\sin\left(\dfrac{1}{x}\right)\cdot e^{\frac{1}{x}}\,\mathrm{d}\,x$;

Mathcad

$$\int\frac{1}{x^2}\cdot\sin\left(\frac{1}{x}\right)\cdot e^{\frac{1}{x}}\,dx\rightarrow\frac{1}{2}\cdot\exp\left(\frac{1}{x}\right)\cdot\cos\left(\frac{1}{x}\right)-\frac{1}{2}\cdot\sin\left(\frac{1}{x}\right)\cdot\exp\left(\frac{1}{x}\right)$$

Mathematica

```
In[20]:= ∫ 1/x^2 * Sin[1/x] * e^(1/x) dx
Out[20]= 1/2 e^(1/x) (Cos[1/x] - Sin[1/x])
```

9. Calculaţi integrala dublă

a) $\iint\dfrac{xy}{\sqrt{\left(1+x^2+y^2\right)^3}}\,\mathrm{d}\,x\,\mathrm{d}\,y$;

b) $\iint\arcsin\sqrt{x+y}\,\mathrm{d}\,x\,\mathrm{d}\,y$;

11. Să se calculeze integrala triplă:

a) $\iiint z^2 \,dx\,dy\,dz$;

b) $\iiint x^2+y^2+z^2 \,dx\,dy\,dz$;

c) $\iiint xyz \,dx\,dy\,dz$;

d) $\iiint (x+y)z \,dx\,dy\,dz$.

Rezolvarea ecuatiilor si a sistemelor de ecuatii în Mathcad și Mathematica

1. Determinaţi rădăcinile polinomului:

a) $P(X)=X^4-2X^3-X+2$

b) $P(X)=X^4+X^3-10X^2-34X-26$

Mathcad

$$v:=\begin{pmatrix}-26\\-34\\-10\\1\\1\end{pmatrix} \qquad \text{polyroots}(v)=\begin{pmatrix}-1.934-1.391i\\-1.934+1.391i\\-1.142\\4.01\end{pmatrix}$$

Mathematica

```
In[25]:= N[Solve[x^4 + x^3 - 10 * x^2 - 34 * x - 26 == 0, x], 4]
Out[25]= {{x → -1.934 - 1.391 i}, {x → -1.934 + 1.391 i}, {x → -1.142}, {x → 4.010}}
```

c) $P(X)=X^7-2X^6+3X^5-4X^4+5X^3-6X^2+7X-8$.

2. Fiind dat polinomul $P(X)=X^4+X^3+X^2+X+1$ să se calculeze suma:

$$S=\frac{1}{x_1-2}+\frac{1}{x_2-2}+\frac{1}{x_3-2}+\frac{1}{x_4-2},$$

unde x_i, $i=\overline{1,4}$ sunt rădăcinile ecuaţiei $P(x)=0$.

3. Rezolvaţi ecuaţiile algebrice:

a) $\dfrac{1}{x-3}+\dfrac{1}{x-1}+\dfrac{2x^2}{(x-3)(x-1)}=0$

Mathcad

$$\frac{1}{x-3}+\frac{1}{x-1}+\frac{2\cdot x^2}{(x-3)\cdot(x-1)}\ \text{solve},x\ \rightarrow -2$$

Mathematica

```
In[22]:= Solve[1/(x - 3) + 1/(x - 1) + (2 * x^2)/((x - 3) * (x - 1)) == 0, x]
Out[22]= {{x → -2}}
```

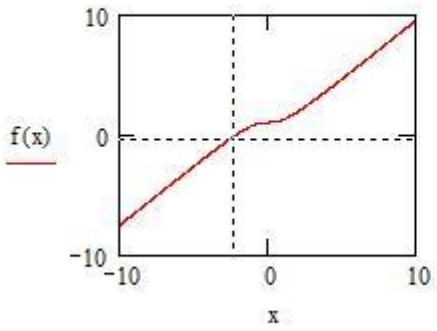
b) $x^2-2(m+2)x+m^2-1=0$ în raport cu variabila x.

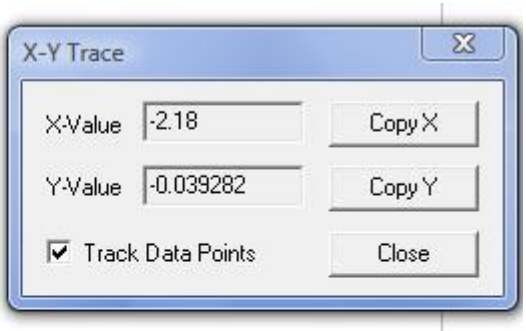
4. Rezolvaţi ecuaţiile transcendente:

a) $1+x=\operatorname{arctg}x$

Mathcad

$f(x) := 1 + x - \operatorname{atan}(x)$





$\text{root}(f(x),x,-3,3) = -2.132268$

Mathematica

```
In[27]:= N[FindInstance[1 + x == ArcTan[x], x, Reals], 7]
Out[27]= {{x → -2.132268}}
```

b) $x-0.2=\ln(1+x)$

5. Rezolvaţi inecuaţiile:

a) $\dfrac{x\sqrt{3}+1}{3x^2+2x\sqrt{3}+3}\le 0$

b) $|x+1|\cdot(|2x+3|-3)<0$

Mathcad

$$|x+1|\cdot(2\cdot|x+3|-3)<0 \text{ solve},x \rightarrow \begin{pmatrix} x<\frac{-3}{2} \\ \frac{-9}{2}<x \end{pmatrix}$$

Mathematica

```
In[3]:= Reduce[Abs[x + 1] * (2 * Abs[x + 3] - 3) < 0, x, Reals]
Out[3]= -9/2 < x < -3/2
```

6. Să se rezolve următoarele sisteme de ecuaţii liniare:

a) $\begin{cases} 6x_1+x_2-x_3+x_4=2 \\ x_1+7x_2+2x_3-x_4=-1 \\ -x_1+2x_2+8x_3+x_4=12 \\ x_1-x_2+x_3+9x_4=-5 \end{cases}$

b) $\begin{cases} 8x_1-x_2-2x_3=2.3 \\ 10x_2+x_3+2x_4=-0.5 \\ -x_1+6x_3+2x_4=-1.2 \\ 3x_1-x_2+2x_3+12x_4=3.7 \end{cases}$

c) $\begin{cases} 14x_1 + x_2 = 13 \\ x_1 + 5x_2 - x_3 = -3 \\ -x_2 + 14x_3 - 2x_4 = -15 \\ -2x_3 + 5x_4 = 7 \end{cases}$

Mathcad

$$A := \begin{pmatrix} 14 & 1 & 0 & 0 \\ 1 & 5 & -1 & 0 \\ 0 & -1 & 14 & -2 \\ 0 & 0 & -2 & 5 \end{pmatrix} \qquad b := \begin{pmatrix} 13 \\ -3 \\ -15 \\ 7 \end{pmatrix}$$

$$x := \text{lsolve}(A, b) \qquad x = \begin{pmatrix} 1 \\ -1 \\ -1 \\ 1 \end{pmatrix}$$

Mathematica

```
In[6]:= Solve[{14 * x1 + x2 == 13, x1 + 5 * x2 - x3 == -3, -x2 + 14 * x3 - 2 * x4 == -15, -2 * x3 + 5 * x4 == 7}, {x1, x2, x3, x4}]
Out[6]= {{x1 → 1, x2 → -1, x3 → -1, x4 → 1}}
```

sau

```
In[7]:= A = {{14, 1, 0, 0}, {1, 5, -1, 0}, {0, -1, 14, -2}, {0, 0, -2, 5}}; b = {{13}, {-3}, {-15}, {7}};

In[12]:= LinearSolve[A, b]
Out[12]= {{1}, {-1}, {-1}, {1}}
```

7. Determinaţi matricea A astfel încât:

$$(2A - 3\cdot(1 \quad 2 \quad 0))^T = 3A^T + (2 \quad 1 \quad -1)^T.$$

Pasul 1. Fie $A = (a \quad b \quad c)$.

Pasul 2. Calculăm $X = (2A - 3\cdot(1 \quad 2 \quad 0))^T - 3A^T - (2 \quad 1 \quad -1)^T$.

Pasul 3. Determinăm a, b, c astfel încât $X = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$.

Mathcad

$$A(a,b,c) := \begin{pmatrix} a \\ b \\ c \end{pmatrix}$$

$$U(a,b,c) := 2 \cdot A(a,b,c) - 3 \cdot \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} - 3 \cdot A(a,b,c) - \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$$

$$U(a,b,c)\ \text{solve}, \begin{pmatrix} a \\ b \\ c \end{pmatrix} \rightarrow (-5 \quad -7 \quad 1)$$

Mathematica

```
In[1]:= A = {a, b, c};

In[3]:= U = 2 * A - 3 * {1, 2, 0} - 3 * A - {2, 1, -1}
Out[3]= {-5 - a, -7 - b, 1 - c}

In[19]:= Solve[U == {0, 0, 0}, {a, b, c}]
Out[19]= {{a → -5, b → -7, c → 1}}
```

8. Rezolvaţi sistemele neliniare:

a) $\begin{cases} x - \sin(x+y) = 0 \\ y - \cos(x-y) = 0 \end{cases}$ (considerând ca punct iniţial $(0,1)$)

Mathcad

$x := 3 \qquad y := 1$

Given

$x - \sin(x+y) = 0 \qquad y - \cos(x-y) = 0$

$$\text{find}(x,y) = \begin{pmatrix} 0.935 \\ 0.998 \end{pmatrix}$$

Mathematica

```
In[13]:= FindRoot[{x - Sin[x + y] == 0, y - Cos[x - y] == 0}, {{x, 3}, {y, 1}}]
Out[13]= {x → 0.935082, y → 0.99802}
```

b) $\begin{cases} x_1 + 3\lg x_1 - x_2^2 = 0 \\ 2x_1^2 - x_1 x_2 - 5x_1 + 1 = 0 \end{cases}$ (considerând ca punct iniţial $(3,2)$)

c) $\begin{cases} x^2 + y^2 + z^2 = 0 \\ 2x^2 + y^2 - 4z = 0 \\ 3x^2 - 4y + z^2 = 0 \end{cases}$ (considerând ca punct iniţial $(0.5, 0.5, 0.5)$)

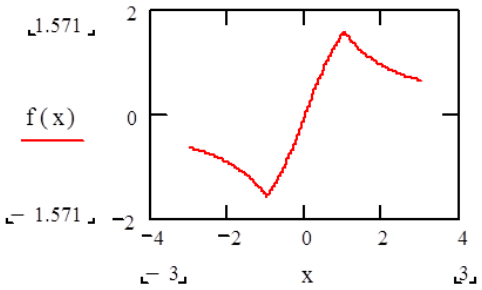
Laborator 3. Reprezentări grafice 2D în Mathcad și Mathematica

1. Să se reprezinte grafic următoarele funcţii

a) $f(x)=\arcsin\dfrac{2x}{1+x^2}$, $x\in[-3,3]$, $h=0.001$; b) $f(x)=\left(\dfrac{x+5}{\ln(x+5)}\right)^{\frac{1}{2}}$, $x\in[-3,5]$;

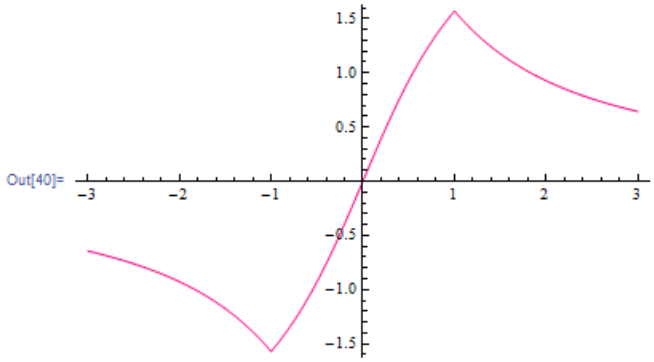
Mathcad

$$f(x) := \operatorname{asin}\left(\frac{2\cdot x}{1+x^2}\right) \qquad x := -3, -2.999 .. 3$$



Mathematica

```
In[32]:= f[x_] := ArcSin[2 * x / (1 + x^2)];
In[40]:= Plot[f[x], {x, -3, 3}, PlotPoints → 6000]
```

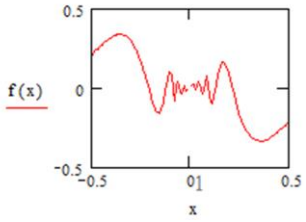


c) $f(x)=\begin{cases} x\cos\dfrac{1}{x}, & x\neq 0 \\ 0, & x=0 \end{cases}$, $x\in[-0.5,0.5]$, $h=0.01$

Mathcad

$$f(x) := \begin{cases} x\cdot\cos\left(\frac{1}{x}\right) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

$$x := -0.5, -0.49 .. 0.5$$

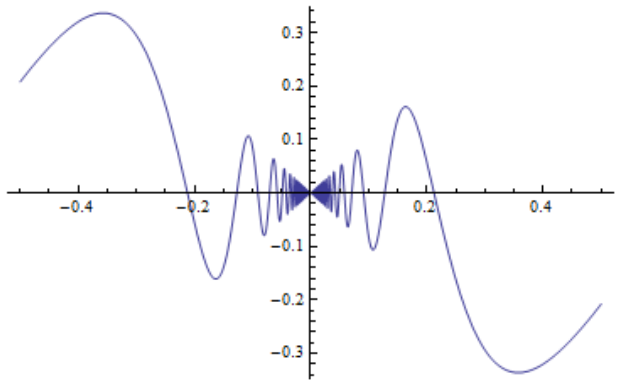


Mathematica

```
In[49]:= g[x_] := { x * Cos[1/x]   x != 0
                  { 0              x == 0

In[50]:= Plot[g[x], {x, -0.5, 0.5}, PlotPoints → 100]
```

Out[50]=

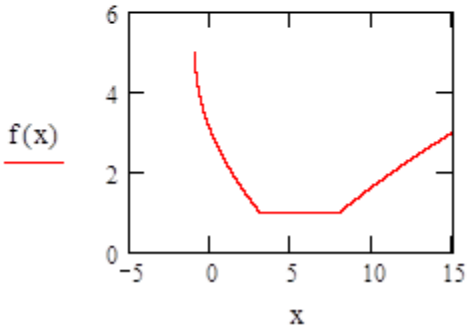


d) $f(x)=\begin{cases} -2\sqrt{x+1}+5, & -1\le x\le 3 \\ 1, & 3<x<8 \\ 2\sqrt{x+1}-5, & 8\le x\le 15 \end{cases}$

Mathcad

$$f(x) := \begin{cases} -2\sqrt{x+1}+5 & \text{if } -1 \le x \le 3 \\ 1 & \text{if } 3 < x < 8 \\ 2\sqrt{x+1}-5 & \text{if } 8 \le x \le 15 \end{cases}$$

$$x := -1, -0.999 .. 15$$



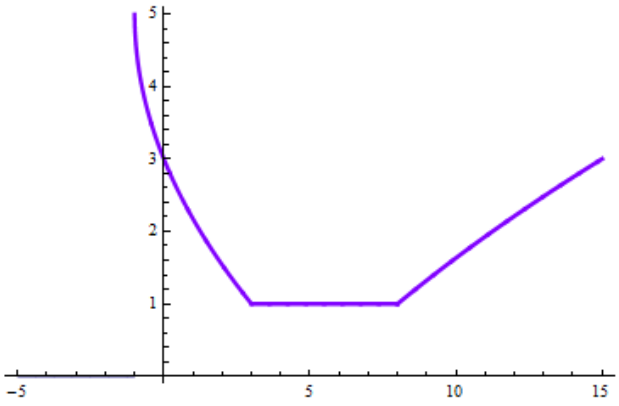
Mathematica

```
In[51]:= Clear[f];

In[52]:= f[x_] := { (-2) * Sqrt[x + 1] + 5   -1 ≤ x ≤ 3
                    1                        3 < x < 8
                    2 * Sqrt[x + 1] - 5      8 ≤ x ≤ 15

In[68]:= Plot[f[x], {x, -5, 15}, PlotPoints → 16000]
```

Out[68]=

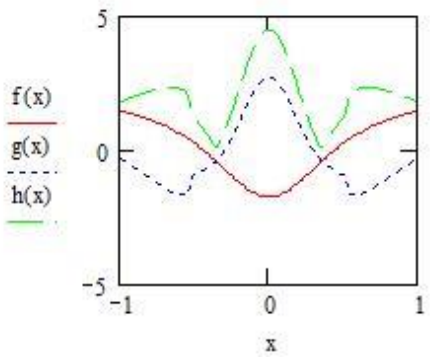


e) $$\begin{cases} f(x) = \dfrac{\left|x^2-2\right| - \left|x^2-3\right|}{x^2+\dfrac{1}{4}} + \sqrt{5} \\ g(x) = f(x)^2 + \sqrt[3]{\cos 3x} - \sqrt{2}\,, \quad x \in [-1,1],\ h = 0.02 \\ h(x) = \left|f(x) - g(x)\right| \end{cases}$$

Mathcad

$x := -1, -0.98 .. 1$

$$f(x) := \frac{\left|x^2-2\right| - \left|x^2-3\right|}{x^2+\frac{1}{4}} + \sqrt{5} \qquad g(x) := f(x)^2 + \sqrt[3]{\cos(3x)} - \sqrt{2} \qquad h(x) := \left|f(x) - g(x)\right|$$



Mathematica

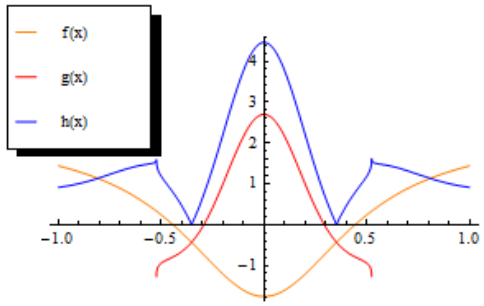
```
In[39]:= Clear[f, g]

In[71]:= f[x_] := (Abs[x^2 - 2] - Abs[x^2 - 3])/(x^2 + 1/4) + Sqrt[5]

In[72]:= g[x_] := f[x]^2 + (Cos[3*x])^(1/3) - Sqrt[2]

In[81]:= h[x_] := Abs[f[x] - g[x]];
         Needs["PlotLegends`"]

In[84]:= Plot[{f[x], g[x], h[x]}, {x, -1, 1}, PlotPoints → 100, PlotLegend → {"f(x)", "g(x)", "h(x)"}, PlotStyle → {Orange, Red, Blue}]
```



f) $f(x)=\arcsin x,\ g(x)=\arccos x,\ x\in[-1,1]$

2. Reprezentaţi grafic următoarele curbe remarcabile din geometrie:

a) *Cercul* are ecuaţia carteziană implicită: $(x-x_0)^2+(y-y_0)^2=r^2$ şi ecuaţiile parametrice:

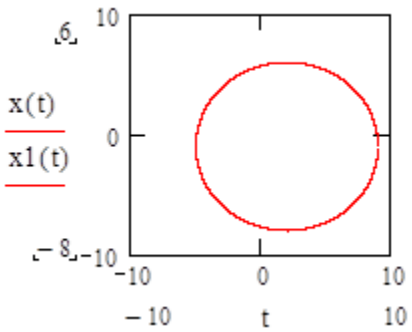
$$\begin{cases} x=x_0+r\cos t \\ y=y_0+r\sin t \end{cases},\ t\in[0,2\pi].$$

Mathcad

$xc := 2 \qquad yc := -1 \qquad r := 7$

$x(t) := \sqrt{r^2-(t-xc)^2}+yc \qquad x1(t) := -\sqrt{r^2-(t-xc)^2}+yc$

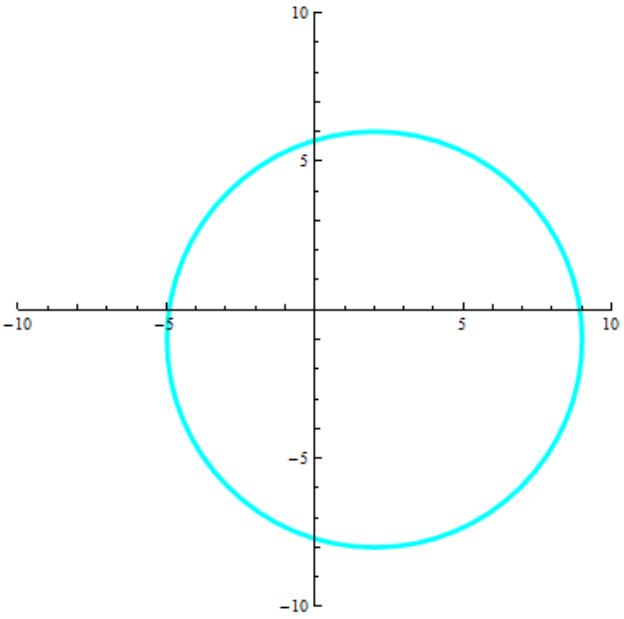
$t := xc - r, xc - r + 0.001 .. xc + r$



Mathematica

```
In[79]:= Graphics[Circle[{2, -1}, 7], Axes → True, PlotRange → {{-10, 10}, {-10, 10}}]
```

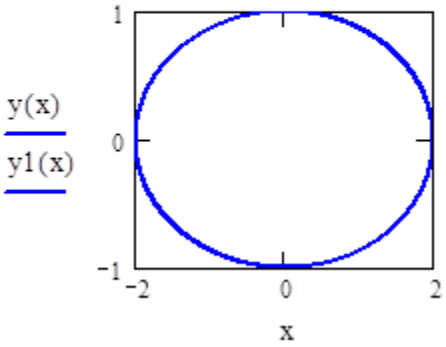
Out[79]=



b) *Elipsa* are ecuaţia carteziană implicită: $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1,\ a>b>0$ şi este caracterizată de ecuaţiile parametrice: $\begin{cases} x = a\cos\theta \\ y = b\sin\theta \end{cases},\ \theta\in[0,2\pi].$

Mathcad

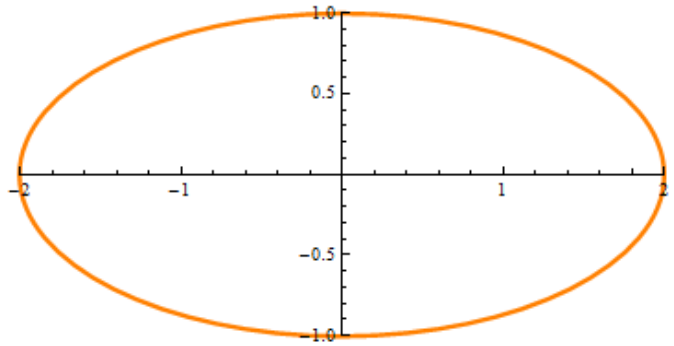
$a := 2 \quad b := 1 \quad x := -a, -a + 0.001 .. a \quad y(x) := \frac{b}{a}\cdot\sqrt{a^2 - x^2} \quad y1(x) := \frac{-b}{a}\cdot\sqrt{a^2 - x^2}$



Mathematica

```
In[82]:= Graphics[Circle[{0, 0}, {2, 1}], Axes → True, PlotRange → {{-2, 2}, {-1, 1}}]
```

Out[82]=



c) *Parabola* are ecuaţia cartezianã implicită: $y^2 = \begin{cases} 2px, & x > 0 \\ -2px, & x \le 0 \end{cases}$ şi ecuaţiile parametrice:

$$\begin{cases} x = \dfrac{t^2}{2p}, t \in \Re \\ y = t \end{cases}.$$

d) *Hiperbola* are ecuaţia cartezianã implicită:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

şi este caracterizată de ecuaţiile parametrice:

$$\begin{cases} x = a \operatorname{ch} t \\ y = b \operatorname{sh} t \end{cases}, t \in \Re, \text{ daca } x \in (a, \infty]$$

si

$$\begin{cases} x = -a \operatorname{ch} t \\ y = -b \operatorname{sh} t \end{cases}, t \in \Re, \text{ daca } x \in (-\infty, -a].$$

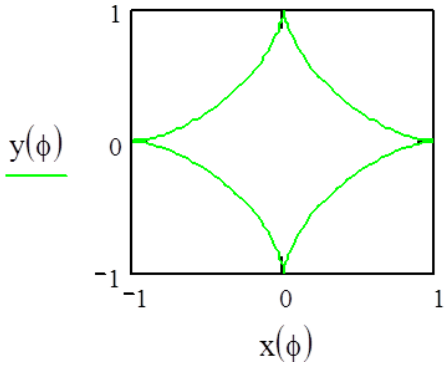
e) *Astroida* are ecuaţia cartezianã implicită: $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$ şi ecuaţiile parametrice:

$$\begin{cases} x = a \cos^3 t \\ y = a \sin^3 t \end{cases}, t \in [0, 2\pi]$$

Mathcad

$a := 1 \qquad \phi := 0, 0.01 \cdot \pi \,..\, 2 \cdot \pi$

$x(\phi) := a \cdot \cos(\phi)^3 \qquad y(\phi) := a \cdot \sin(\phi)^3$

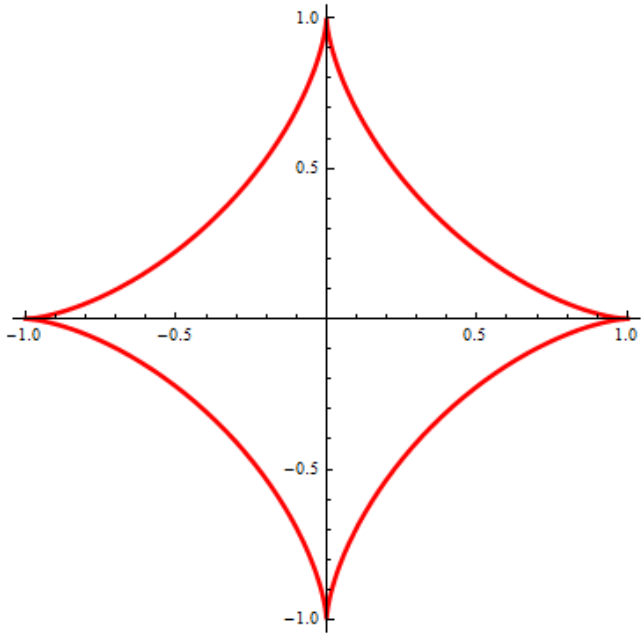


Mathematica

```
In[83]:= a = 1;

In[87]:= ParametricPlot[{a * Cos[ϕ]^3, a * Sin[ϕ]^3}, {ϕ, 0, 2 * π}, PlotPoints → 200]
```

Out[87]=



f) *Cardioida* are ecuaţiile parametrice: $\begin{cases} x = a\cos t(1-\cos t) \\ y = a\sin t(1-\cos t) \end{cases}, t \in [0, 2\pi].$

g) *Lemniscata lui Bernoulli* are ecuaţiile parametrice:

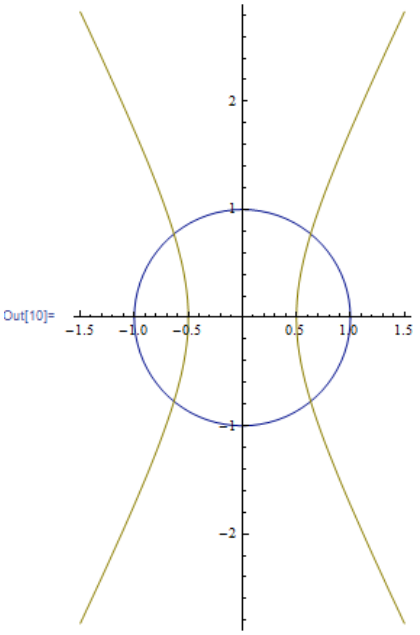
$$\begin{cases} x = \dfrac{a\cos\varphi}{1+\sin^2\varphi} \\ y = \dfrac{a\sin\varphi\cos\varphi}{1+\sin^2\varphi} \end{cases}, \quad \varphi \in [0, 2\pi].$$

h) *Curba Butterfly* : $\begin{cases} x = \left(e^{\cos t} - 2\cos 4t + \sin^5 \frac{t}{12} \right) \cdot \sin t \\ y = \left(e^{\cos t} - 2\cos 4t + \sin^5 \frac{t}{12} \right) \cdot \cos t \end{cases}, t \in [0, 2\pi].$

3. Reprezentati grafic cercul $x^2 + y^2 = 1$ si hiperbola $4x^2 - y^2 = 1$ pentru $-1.5 \le x \le 1.5.$

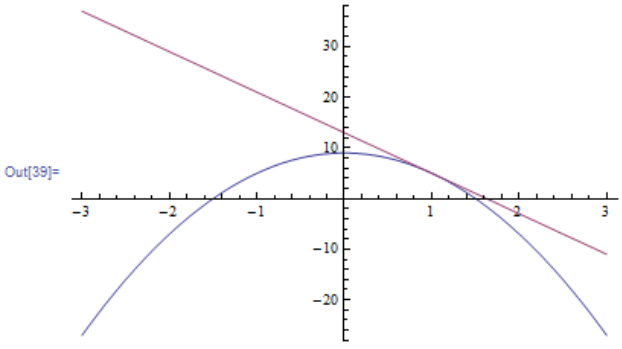
Mathematica

```
In[9]:= Needs["Graphics`ImplicitPlot`"]
In[10]:= ImplicitPlot[{x^2 + y^2 == 1, 4*x^2 - y^2 == 1}, {x, -1.5, 1.5}]
```



4. Reprezentati grafic tangenta la graficul functiei $f(x) = 9 - 4x^2$ în punctul $(1, f(1))$.

```
In[34]:= f[x_] := 9 - 4*x^2
In[39]:= Plot[{f[x], f'[1]*(x - 1) + f[1]}, {x, -3, 3}]
```

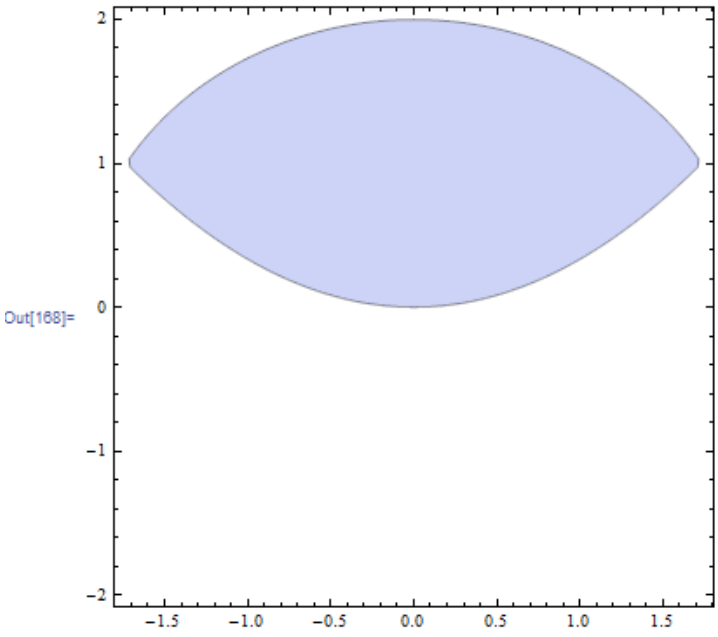


5. Reprezentati grafic regiunea definită de inegalitatile: $x^2 + y^2 \le 4,\ 3y \ge x^2$.

Mathematica

```
In[168]:= RegionPlot[x^2 + y^2 ≤ 4 && 3 * y ≥ x^2, {x, -√3, √3}, {y, -2, 2}]
```

Out[168]=

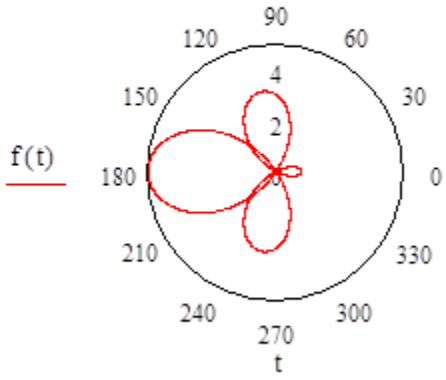


6. Reprezentaţi grafic în coordonate polare următoarele funcţii:

a) *Scarabaeus*: $f(t) = b\cos 2t - a\cos t,\ t \in [0, 2\pi]$

Mathcad

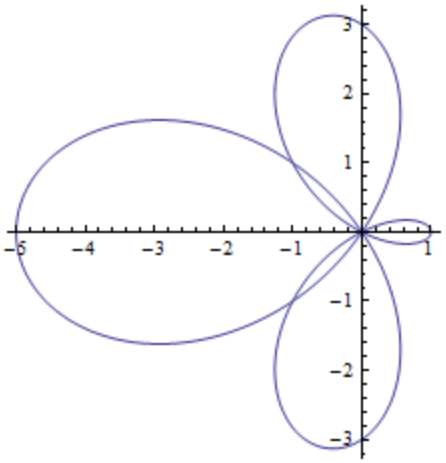
$a := 2 \quad b := 3 \quad t := 0, 0.001 \cdot \pi .. 2 \cdot \pi \quad f(t) := b \cdot \cos(2 \cdot t) - a \cdot \cos(t)$



Mathematica

```
In[92]:= PolarPlot[3 * Cos[2 * t] - 2 * Cos[t], {t, 0, 2 * Pi}, PlotPoints → 2000]
```

Out[92]=



b) *Cardioida*: $f(\phi) = a(1-\cos\phi),\ \phi \in [0, 2\pi]$.

c) *Trifoiul cu patru foi*: $f(t) = a\sin 2t,\ t \in [0, 2\pi]$.

Laborator 4. Reprezentări grafice 3D în Mathcad și Mathematica

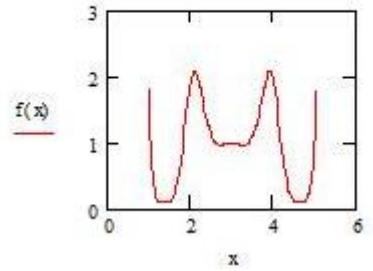
1. Reprezentați grafic suprafața obținută prin rotirea curbei, care are ecuația

a) $f(x)=e^{-(x-3)^2}\cos 4(x-3)$, $x\in[1,5]$ în jurul axei Ox.

b) $g(x)=x^2\sin x$, $x\in[0,\pi]$ în jurul axei Ox și respectiv în jurul axei Oy.

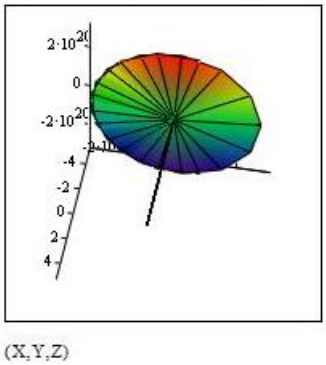
Mathcad

$f(x) := e^{-(x-3)^2\cdot\cos[4\cdot(x-3)]}$ $\quad x := 1, 1.01 .. 5$



$0 \le t \le 2\cdot\pi$ $\quad 1 \le r \le 5$

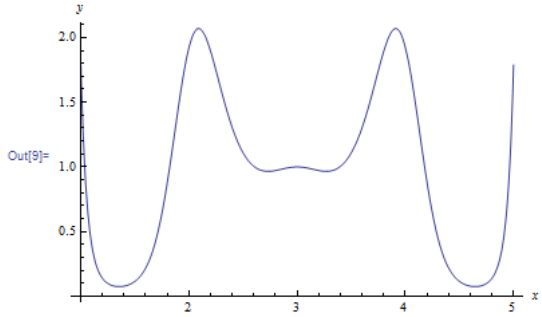
$X(r,t) := r$ $\quad Y(r,t) := f(r)\cdot\cos(t)$ $\quad Z(r,t) := f(r)\cdot\sin(t)$



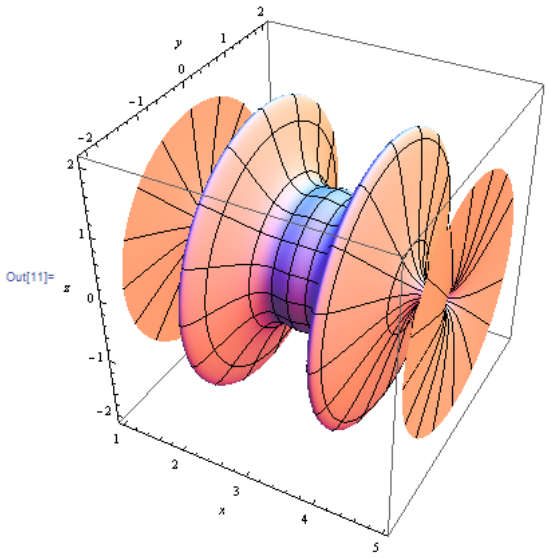
Mathematica

```
In[8]:= f[x_] := e^-(x-3)^2*Cos[4*(x-3)]

In[9]:= Plot[f[x], {x, 1, 5}, PlotPoints → 1000, AxesLabel → {x, y}]
```




```
In[11]:= ParametricPlot3D[{r, f[r]*Cos[t], f[r]*Sin[t]}, {r, 1, 5}, {t, 0, 2*π}, PlotPoints → {100, 100}, AxesLabel → {x, y, z}]
```



sau

```
In[32]:= RevolutionPlot3D[f[x], {x, 1, 5}, RevolutionAxis → {1, 0, 0}]
```

2. Reprezentaţi grafic în 3D următoarele suprafeţe:

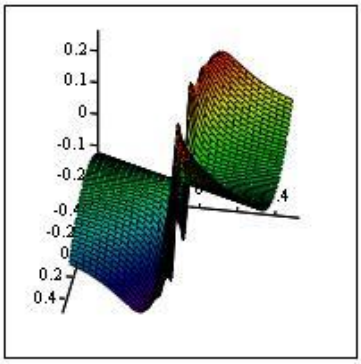
a) $f(x,y)=\dfrac{x^2y}{x^4+4y^2}$, $x,y\in[-1/2,-1/2]$

Mathcad

$$f(x,y):=\frac{x^2\cdot y}{x^4+4\cdot y^2}$$

$$x0:=\frac{-1}{2}\quad x1:=\frac{1}{2}\quad y0:=\frac{-1}{2}\quad y1:=\frac{1}{2}$$

c := CreateMesh(f,x0,x1,y0,y1,50,30)



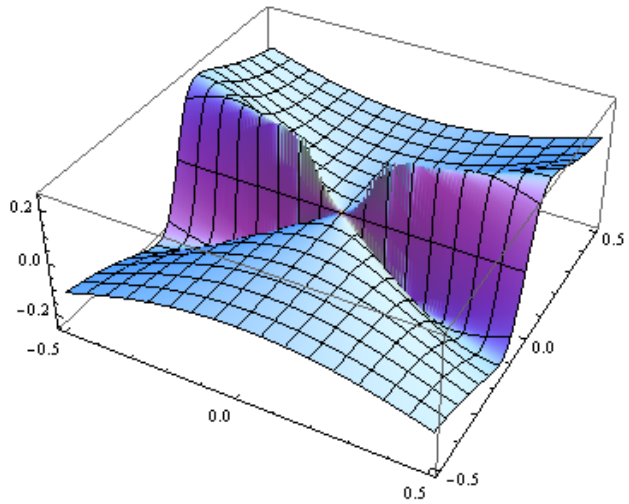
c

Mathematica

```
In[58]:= f[x_, y_] := x^2 * y / (x^4 + 4 * y^2)

In[62]:= Plot3D[f[x, y], {x, -1/2, 1/2}, {y, -1/2, 1/2}, PlotPoints → {50, 30}]
```

Out[62]=



b) $f(x,y)=x^2+x^2y^2+y^4,\ x,y\in[-1,1]$

c) $f(x,y)=6x^2y-3x^4-2y^3,\ x\in[-2,2],\ y\in[-2,3];$

d) $f(x,y)=(y-1)^2e^{-(x+1)^2-y^2}\ -\frac{10}{3}\left(-x^5+\frac{1}{5}y-y^3\right)e^{-x^2-y^2}\ -\frac{1}{9}e^{-x^2-(y+1)^2}$,

$x,y\in[-3,3];$

e) $f(x,y)=x\sin y+y\sin x,\ x,y\in[0,2\pi];$

3. Reprezentați grafic regiunea situată între suprafețele $z=4-x^2-y^2$ și $z=2-x$, pentru $x\in[-2,2]$, $y\in[-2,2]$.

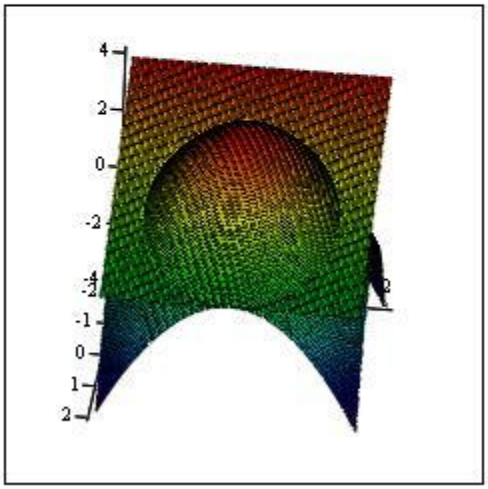
Mathcad

$f(x,y) := 4 - x^2 - y^2$ $\quad$ $g(x,y) := 2 - x$

$x0 := -2$ $\quad$ $x1 := 2$ $\quad$ $y0 := -2$ $\quad$ $y1 := 2$

$c := \text{CreateMesh}(f, x0, x1, y0, y1, 100, 100)$

$c1 := \text{CreateMesh}(g, x0, x1, y0, y1, 100, 100)$



c, c1

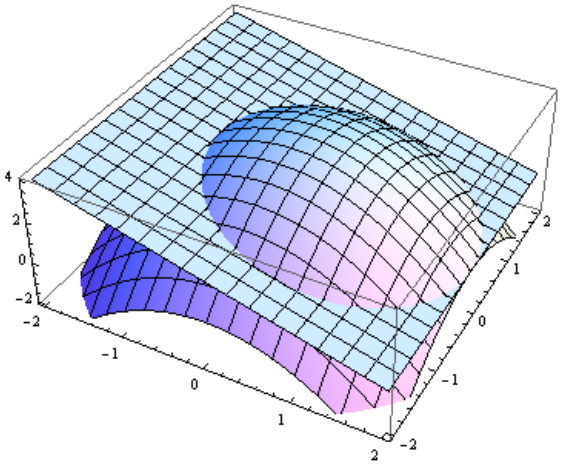
Mathematica

```
In[52]:= p1 = Plot3D[4 - x^2 - y^2, {x, -2, 2}, {y, -2, 2}, PlotPoints → 100];

In[53]:= p2 = Plot3D[2 - x, {x, -2, 2}, {y, -2, 2}, PlotPoints → 100];

In[55]:= Show[p1, p2, PlotRange → {{-2, 2}, {-2, 2}, {-2, 4}}]
```

Out[55]=

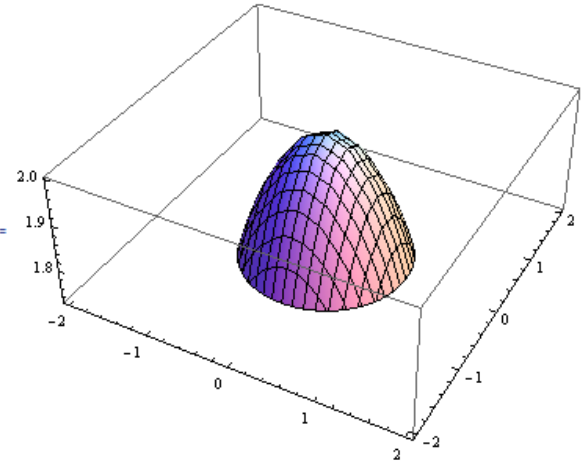


4. Reprezentați grafic porțiunea din suprafața $f(x,y)=\sqrt{4-x^2-y^2}$, situată în regiunea $R=\{(x,y)|\ x^2+y^2\le 1\}$

Mathematica

```
In[152]:= Plot3D[Sqrt[4 - x^2 - y^2], {x, -2, 2}, {y, -2, 2}, RegionFunction → Function[{x, y, z}, x^2 + y^2 ≤ 1]]
```

Out[152]=



5. Reprezentați grafic următoarele cuadrice:

a) *Sfera:* are ecuația în coordonate carteziene: $x^2+y^2+z^2=R^2$ și reprezentarea parametrică: $\begin{cases} x=\rho\sin\theta\cos\varphi \\ y=\rho\sin\theta\sin\varphi \\ z=\rho\cos\theta \end{cases}$, $\rho>0$, $\theta\in[0,\pi]$, $\varphi\in[0,2\pi]$.

Mathcad

$r := 2$

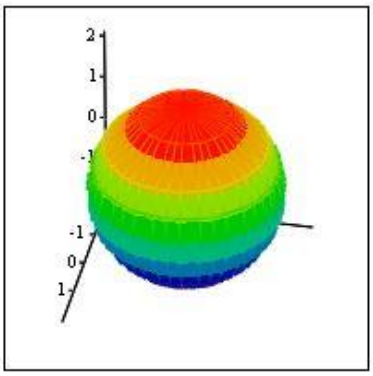
$X(\phi,\theta) := r \cdot \sin(\phi) \cdot \cos(\theta)$

$Y(\phi,\theta) := r \cdot \sin(\phi) \cdot \sin(\theta)$

$Z(\phi,\theta) := r \cdot \cos(\phi)$

$0 \le \phi \le \pi$

$0 \le \theta \le 2 \cdot \pi$

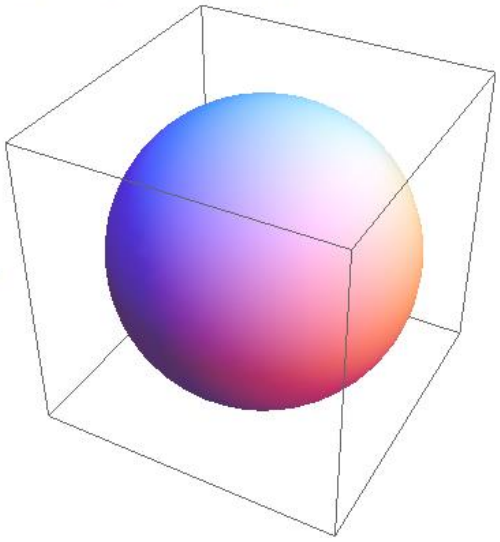


Mathematica

(X,Y,Z)

```
In[67]:= Graphics3D[Sphere[{0, 0, 0}, 2]]
```

Out[67]=



b) *Conul de ordin doi*: are ecuaţia în coordonate carteziene $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 0$, $a \ge b > 0$, $c > 0$

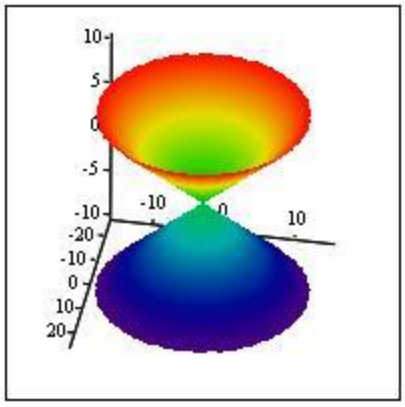
şi reprezentarea parametrică: $\begin{cases} x = av\cos u \\ y = bv\sin u \\ z = \pm cv \end{cases}$, $u \in [0, 2\pi]$, $v \in \Re$.

Mathcad

$a := 5 \quad b := 3 \quad c := 2 \quad 0 \le u \le 2\cdot\pi \quad 0 \le v \le 50$

$X(u,v) := a\cdot v\cdot\cos(u) \quad Y(u,v) := b\cdot v\cdot\sin(u) \quad Z(u,v) := c\cdot v$

$Z1(u,v) := -c\cdot v$



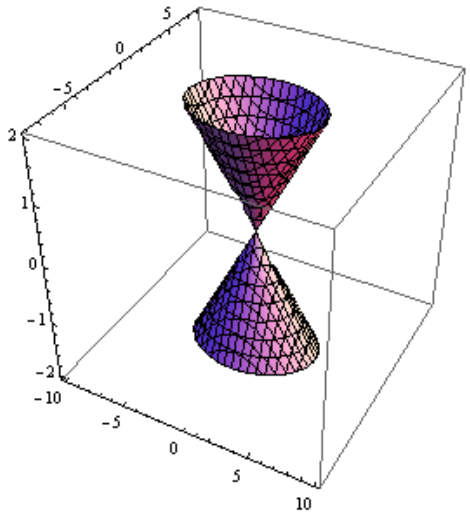
(X,Y,Z),(X,Y,Z1)

Mathematica

In[31]:= `Needs["Graphics`ContourPlot3D`"]`

In[32]:= `ContourPlot3D[x^2/25 + y^2/9 - z^2/4 == 0, {x, -10, 10}, {y, -8, 8}, {z, -2, 2}]`

Out[32]=



c) *Elipsoidul:* are ecuaţia în coordonate carteziene: $\frac{x}{a}^2 + \frac{y}{b}^2 + \frac{z}{c}^2 = 1$

$a \ge b \ge c > 0$ şi reprezentarea parametrică: $\begin{cases} x = a\sin u\cos v \\ y = b\sin u\sin v \\ z = c\cos u \end{cases}$, $u \in [0,\pi]$, $v \in [0,2\pi]$.

d) *Hiperboloidul cu o pânză*: are ecuaţia în coordonate carteziene: $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1,\ a\geq b>0,\ c>0$ şi reprezentarea parametrică:

$$\begin{cases} x = a\sqrt{1+u^2}\cos v \\ y = b\sqrt{1+u^2}\sin v,\ u\in\Re,\ v\in[0,2\pi]. \\ z = cu \end{cases}$$

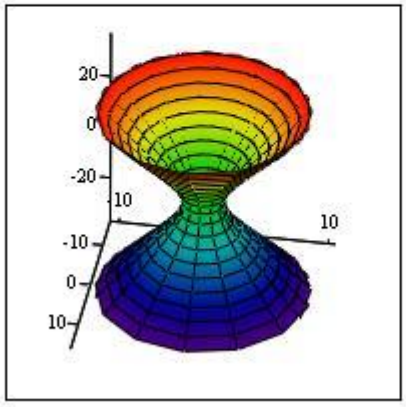
Mathcad

$a := 3 \quad b := 2 \quad c := 7 \qquad 0 \le v \le 2\cdot\pi \qquad -2 \le u \le 2$

$X(u,v) := a\cdot\sqrt{1+u^2}\cdot\cos(v)$

$Y(u,v) := b\cdot\sqrt{1+u^2}\cdot\sin(v)$

$Z(u,v) := c\cdot u$



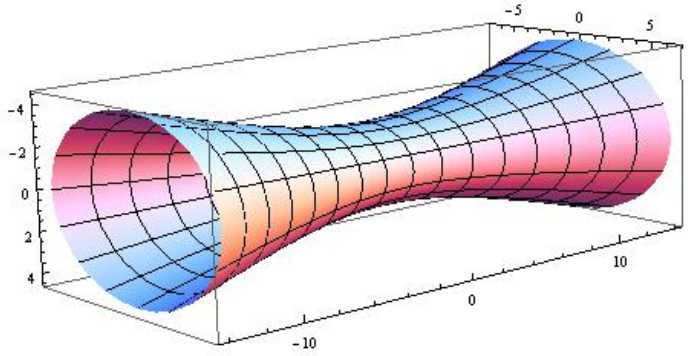
(X,Y,Z)

Mathematica

In[75]:= a = 3; b = 2; c = 7;

In[84]:= ParametricPlot3D[{a*Sqrt[1+u^2]*Cos[v], b*Sqrt[1+u^2]*Sin[v], c*u}, {u, -2, 2}, {v, 0, 2*π}]

Out[84]=



e) *Hiperboloidul cu două pânze*: ecuaţia în coordonate carteziene:

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=-1\ a\geq b>0,\ c>0$$

şi reprez. param: $\begin{cases} x=a\sinh u\cos v \\ y=b\sinh u\sin v \\ z=\pm c\cosh u \end{cases},\ u\in\Re,\ v\in[0,2\pi].$

f) *Paraboloidul hiperbolic*: are ecuaţia în coordonate carteziene: $\frac{x^2}{a^2}-\frac{y^2}{b^2}=2z$,

$a>0,\ b>0$ şi reprezentarea parametrică: $\begin{cases} x=a\sqrt{2v}\cos u \\ y=b\sqrt{2v}\sin u \\ z=v\cos 2u \end{cases},\ u\in[0,2\pi],\ v>0.$

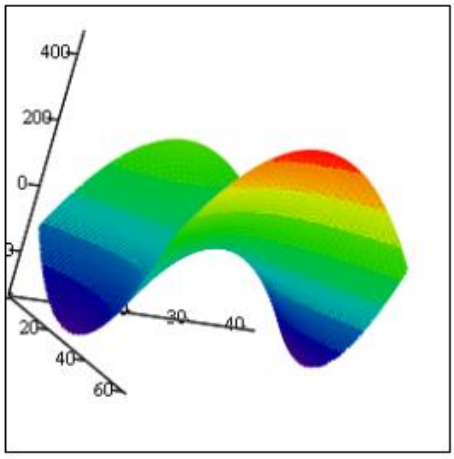
Mathcad

$n := 60$ $a := -60$ $b := 90$ $h := \frac{b-a}{n}$ $k := 0..n$ $x_k := a + k\cdot h$

$aa := 3$ $m := 40$ $c := -50$ $d := 50$ $q := \frac{d-c}{m}$ $j := 0..m$ $y_j := c + j\cdot q$

$bb := 2$ $f(x,y) := \frac{x^2}{aa^2\cdot 2}-\frac{y^2}{bb^2\cdot 2}$

$M_{k,j} := f(x_k, y_j)$



M

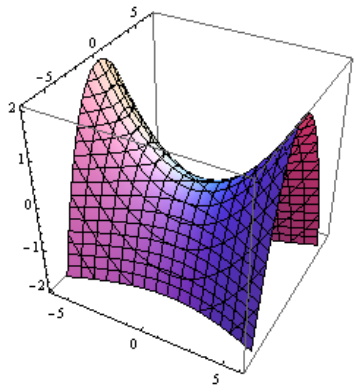
Mathematica

```
In[11]:= Needs["Graphics`ContourPlot3D`"]

In[25]:= a := 3; b := 2;

In[26]:= ContourPlot3D[x^2/a^2 - y^2/b^2 == 2*z, {x, -6, 6}, {y, -7, 7}, {z, -2, 2}]
```

Out[26]=



g) *Paraboloidul eliptic:* are ecuaţia în coordonate carteziene $\frac{x^2}{a^2}+\frac{y^2}{b^2}=2z$,

$a>0,\ b>0$ şi reprezentarea parametrică: $\begin{cases} x=a\sqrt{2v}\cos u \\ y=b\sqrt{2v}\sin u \\ z=v \end{cases},\ u\in[0,2\pi],\ v>0.$

6. Reprezentaţi grafic regiunea marginită de: $\begin{cases} x^2+y^2\geq az \\ x^2+y^2\leq a^2 \\ z\geq 0 \end{cases}$

Mathematica

```
In[149]:= a = 7;

In[150]:= RegionPlot3D[x^2 + y^2 ≥ a*z && x^2 + y^2 ≤ a^2 && z ≥ 0, {x, -7, 7}, {y, -7, 7}, {z, -2, 2}, PlotPoints → 35]
```

Out[150]=